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Dispersion Analysis by a Single Application of Simultaneous Tolerances

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Engineering Science Operations

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AIR FORCE SYSTEMS COMMAND
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DISPERSION ANALYSIS BY A SINGLE APPLICATION OF SIMULTANEOUS TOLERANCES

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FOREWORD

This report is published by Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-68-C-0200.

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

This paper presents a new and simplified method of performing a dispersion analysis. The technique consists of performing a single analysis in which appropriately scaled system tolerances are applied simultaneously in the same direction. This contrasts with the popular and well-known root-sum-square approach, which requires conducting a series of analyses in which the system parameters are varied one at a time and the results are then root-sum-squared. An interesting corollary of the technique is the estimation of the number of standard deviations contained in a dispersion resulting from a number of parameters being displaced from their nominal values in a worst-on-worst fashion.

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SYMBOLS

K_i	ith influence coefficient of the function Y (partial derivative of Y with respect to X_i)
k	number of standard deviations contained in the dispersion ΔY due to the statistical combination of $k\sigma$ dispersions in the X_i parameters
m	number of standard deviations contained in the ΔX_i dispersions when applying the simultaneous tolerance technique (may be interpreted as a scaling factor on the 1σ dispersions in the X_i parameters)
X_i	ith parameter of the function Y, which is subject to tolerance deviations
ΔX_i	incremental change in X_i due to a perturbation from nominal
Y	function being investigated for the effect of tolerances (is a function of the parameters $X_1, X_2, \ldots, X_n$)
ΔY	dispersion in Y due to perturbations in the X_i parameters
μ	mean of the $\lvert K_i \sigma_{X_i} \rvert$ set of tolerance terms
V	coefficient of variation (defined as the ratio of the standard deviation to the mean of a statistical sample)

σ	standard deviation of the $\lvert K_i \sigma_{X_i} \rvert$ set of tolerance terms
σ_{X_i}	standard deviation of the distribution of ΔX_i
σ_Y	standard deviation of the distribution of ΔY

SECTION I

INTRODUCTION

System designs are usually optimized on the basis of nominal parameters; then, some kind of dispersion analysis is performed to verify that the system performs satisfactorily under toleranced conditions. Of the two well-known techniques for accomplishing a tolerance study, i.e., the Monte Carlo approach and the Root-Sum-Square (RSS) method, the latter is by far the more popular for the simple reason that the former is often too costly. Because of this, only the RSS approach will be discussed in this paper as a basis for comparison with the technique proposed herein.

Normally the RSS method is conducted by first calculating a nominal case and then successively varying one parameter at a time, while observing the deviation from nominal caused by the perturbation of that particular parameter. These deviations from nominal are then root-sum-squared to obtain the expected resultant deviation under toleranced conditions.

In this paper it is shown that, under most conditions, it is feasible to perform a single analysis in which the various parameters are toleranced simultaneously, and accomplish essentially the same end result that the RSS method yields.

SECTION II

CONVENTIONAL ROOT-SUM-SQUARE (RSS) APPROACH

Assume that the function Y, which is being investigated for the effect of tolerances, is a function of the parameters $X_1, X_2, \ldots, X_n$. The approximate dispersion in Y due to parameter variations can be expressed by the linear expansion

$$\Delta Y = \sum_{i=1}^{n} K_i \Delta X_i \qquad (1)$$

where the K_i terms are usually referred to as "influence coefficients" and the ΔX_i terms represent the parameter deviations from nominal.

Since the X_i parameters are subject to tolerances, they can be considered as random variables which are statistically distributed about their nominal values. If the X_i parameters are independent, then the k-sigma dispersion in ΔY is given by the familiar RSS relationship

$$k\sigma_Y = \left[\sum_{i=1}^{n} \left(K_i k\sigma_{X_i}\right)^2\right]^{1/2} \qquad (2)$$

where the σ_{X_i} values represent the standard deviations of the individual ΔX_i distributions.

To apply Eq. (2), it is first necessary to evaluate the K_i influence coefficients. Normally, the relationship between Y and its parameters is known only implicitly; therefore, the influence coefficients cannot be evaluated analytically simply by taking the partial derivatives. Instead, use is made of Eq. (1) where the parameters are varied one at a time and an analysis is performed to determine the corresponding influence coefficient for each parameter variation. The important point here is that this procedure requires n separate analyses for n given parameters.

Once $k\sigma_Y$ has been evaluated it is customary to relate it to a probability of success figure, thereby completing the analysis. This can be accomplished in elegant fashion if the distribution of ΔY turns out to be Gaussian. To be assured of a Gaussian distribution for ΔY, the assumption is usually made that the X_i parameters are Gaussian in nature. As such, the resultant distribution of ΔY will also be Gaussian, since a linear combination of normally distributed variables is itself normally distributed. It should also be noted that even if the individual terms comprising ΔY are not Gaussian, the Central Limit Theorem states that the combination of a large number of these terms will approach a Gaussian distribution.

SECTION III

PROPOSED APPROACH

A. ASSUMPTIONS

In addition to the usual assumptions of linearity and statistical independence required for the RSS method, the proposed approach is predicated on the following assumptions:

1. The directional characteristics of the tolerance effects, i.e., the signs associated with the products of the influence coefficients and their corresponding tolerances, are known.
2. The significant or strong tolerance effects outnumber the insignificant or weak tolerance effects.

The first assumtion is vital to the proposed dispersion analysis technique since it depends on applying the tolerances such that their effects are all in the same direction. Satisfaction of this assumption may require some previous experience or some *a priori* reasoning based on the physics of the problem. For example, in the case of booster applications, it is generally true that a thrust increase leads to an increase in burnout velocity, that a decrease in wind magnitude leads to a decrease in vehicle loads, that an increase in autopilot gains leads to a decrease in stability margins, and so on.

Should the problem be such that the directional characteristics of the tolerances cannot be predetermined without extensive analysis, then the obvious choice would be the more laborious RSS method. It is noted, however,

that once this is done the experience gained could then be used to apply the proposed method in succeeding problems of similar nature. For example, optimization studies which require dispersion analyses about different sets of nominal conditions could utilize the RSS method for one set of nominals and, having established the directional characteristics of the tolerances, could employ the proposed method for the remaining sets of nominals.

The second assumption is aimed at achieving good accuracy since the accuracy of the method depends on constraining the value of a statistical parameter known as the coefficient of variation* between zero and unity. Should a preponderance of small terms be toleranced in proportion to the significant terms, then it is possible for the coefficient of variation to exceed unity--thereby diminishing the accuracy of the method.

From a practical standpoint, however, this assumption will tend to be satisfied automatically as the analyst will not (knowingly) include mostly negligible effects. In fact, quite the contrary is true; negligible effects are usually excluded from the analysis.

B. DESCRIPTION OF THE METHOD

The approach suggested in this paper is to perform a single analysis and obtain essentially the same result for $k\sigma_Y$ that would normally be obtained from the conventional RSS approach using Eq. (2). The procedure is simply to apply appropriately scaled tolerances simultaneously, in the proper direction, and note the resultant dispersion in Y due to the combined effect of the tolerances. This is discussed further in the following paragraphs.

*Defined as the ratio of the standard deviation to the mean of a statistical sample. See H. Cramer, Mathematical Methods of Statistics, Princeton University Press, Princeton, New Jersey (1958).

Referring to Eq. (1), it is noted that if all the tolerance effects (products of the influence coefficients and their corresponding tolerances) were in at the same time and in the same direction, say positive, then all the parameter deviations would contribute uniquely to an increase in ΔY. Without this restriction on polarity, some of the tolerance effects could cancel each other and result in a non-unique value for ΔY—the particular value of ΔY depending on the signs associated with the tolerances. Therefore, this particular assumption — that of having all the tolerance effects in the same direction — is vital to the tolerance analysis method proposed in this paper.

Using the above ground rules, the resulting dispersion in Y can be expressed by a modified form of Eq. (1) as

$$\Delta Y = \sum_{i=1}^{n} |K_i \Delta X_i| \qquad (3)$$

where the absolute values denote that all the terms are forced to be positive by proper selections of signs associated with the tolerances.

What we are striving for is a resultant dispersion ΔY that represents the same number of standard deviations k that would be obtained from the conventional RSS analysis. Thus, we can let

$$\Delta Y = k\sigma_Y \qquad (4)$$

Similarly, the ΔX_i tolerances can be assumed to represent a certain number of standard deviations m (as yet undetermined) of their corresponding distributions, that is

$$\Delta X_i = m\sigma_{X_i} \tag{5}$$

Inserting Eqs. (4) and (5) into Eq. (3) yields

$$k\sigma_Y = \sum_{i=1}^{n} |K_i m\sigma_{X_i}| \tag{6}$$

Equation (6) represents the basic philosophy of the proposed tolerance analysis method. The significant thing is that the right hand side of Eq. (6) is obtained directly as the result of a single analysis effort in which all the tolerances are applied simultaneously, whereas the conventional approach requires n analyses to evaluate the individual tolerance effects which are then combined according to Eq. (2).

C. DETERMINATION OF THE TOLERANCE MAGNITUDES

Before the approach indicated by Eq. (6) can actually be utilized, it is first necessary to evaluate the scaling factor m, which determines the magnitudes of the tolerances being applied. Equation (6) can be rewritten slightly by factoring out m to obtain

$$k\sigma_Y = m \sum_{i=1}^{n} |K_i\sigma_{X_i}| \tag{7}$$

Similarly, Eq. (2) can be modified to read

$$k\sigma_Y = k\left[\sum_{i=1}^{n} |K_i\sigma_{X_i}|^2\right]^{1/2} \tag{8}$$

(Here, insertion of the absolute value signs instead of the parentheses is valid because of the squaring process.) Equating Eqs. (7) and (8) and solving for m yields

$$m = \frac{k\left[\sum_{i=1}^{n} |K_i \sigma_{X_i}|^2\right]^{1/2}}{\sum_{i=1}^{n} |K_i \sigma_{X_i}|} \qquad (9)$$

Examination of Eq. (9) indicates that m is a function of an unknown set of numbers representing the tolerance terms $|K_1\sigma_{X_1}|$, $|K_2\sigma_{X_2}|, \ldots,$ $|K_n\sigma_{X_n}|$. Considering the sequence $\{|K_i\sigma_{X_i}|\}$ as a random sequence with a mean

$$\mu = \frac{1}{n} \sum_{i=1}^{n} |K_i \sigma_{X_i}| \qquad (10)$$

and a variance

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} |K_i \sigma_{X_i}|^2 - \mu^2 \qquad (11)$$

and defining the sigma/mean ratio as the coefficient of variation V, Eq. (9) can be simplified to

$$m = k\sqrt{\frac{V^2 + 1}{n}} \qquad (12)$$

where $V \triangleq \frac{\sigma}{\mu} \triangleq$ Coefficient of Variation.

An exact solution for m is actually impossible since it requires a knowledge of the influence coefficients (which are and will remain unknown under the current approach). However, an approximate solution is found which turns out to be surprisingly accurate under most circumstances.

As shown in the Appendix, the coefficient of variation V is generally constrained between the limits of zero and unity, that is

$$0 \leq V \leq 1 \tag{13}$$

Substituting Eq. (13) into Eq. (12) gives the lower and upper bounds on m as

$$\frac{k}{\sqrt{n}} \leq m \leq k\sqrt{\frac{2}{n}} \tag{14}$$

Taking the average of the minimum and maximum values of m as an approximation to the true value of m gives the final result of

$$m \approx 1.2\,\frac{k}{\sqrt{n}} \tag{15}$$

Two interesting observations with regard to Eq. (15) are noted here. First, m is always $<$ k for $n > 1$. Secondly, m decreases as the number of terms increases. Both of these observations make physical sense and lend credibility to Eq. (15).

Finally, it is noted that Eq. (15) can be interpreted two ways. The first way concerns the method of conducting a tolerance analysis that is the subject of this paper. Here k is given and the problem is to find the appropriate scaling on the individual tolerances, i.e., we solve for m.

But, an equally interesting facet of Eq. (15) is the inverse of the above. Suppose it is desired to evaluate the worst case dispersion in ΔY due to the simultaneous stacking of tolerances in a worst-on-worst fashion. Here, m is known and solving for k yields the probability of occurrence associated with this worst case dispersion.

D. ACCURACY

Accuracy tests of the proposed method of simultaneous tolerancing are discussed in this section.

Since the dispersion computed by the proposed method is proportional to the scaling factor m, the error in the dispersion is equal to the error in m itself. The error in m can be computed from the difference between the approximate value of m given by Eq. (15) and the true value given by Eq. (12). In terms of percent error, the result is

$$m_\epsilon = \left(\frac{1.2}{\sqrt{V^2 + 1}} - 1 \right) \times 100\% \tag{16}$$

The error given by Eq. (16) is shown in Fig. 1 as a function of the coefficient of variation V. Note that the error is constrained to the limits of +20% and -15% as long as V is constrained to the limits of 0 and 1, respectively.

To check the accuracy of the method under typical situations, the error in m was computed for a wide variety of one-sided distributions of the $|K_i \sigma_{X_i}|$ array of terms. (For simplicity, the distributions were assumed continuous rather than discrete.) The results are summarized in Table 1. The error in the proposed method of simultaneous tolerancing turns out to be surprisingly

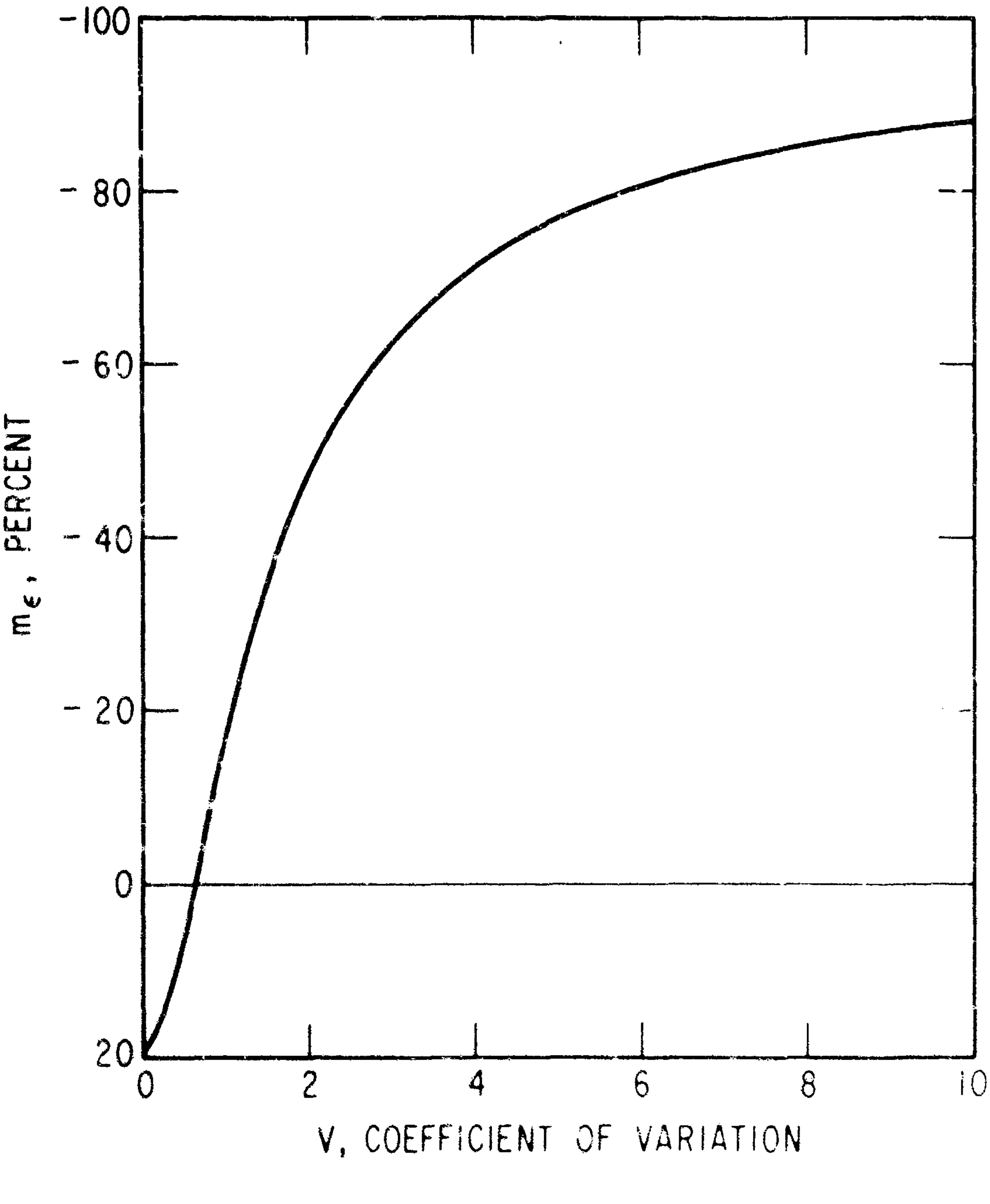
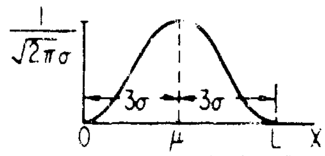
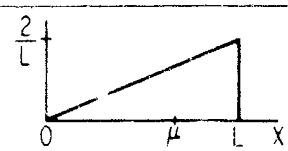
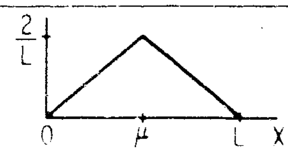
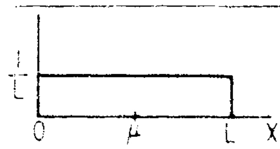
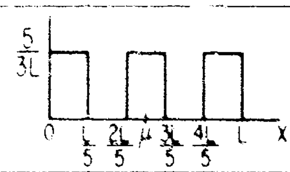
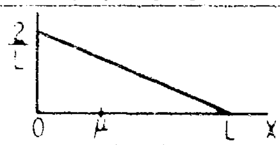
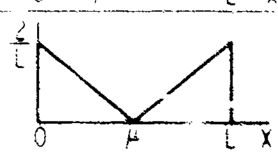
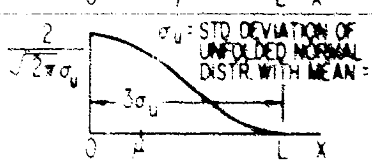
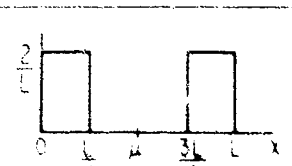


Figure 1. Error in Proposed Tolerance Method as a Function of the Coefficient of Variation

Table 1. Resultant Error in Proposed Tolerance Method for Various Distributions of Component Tolerance Terms

DISTRIBUTION OF X	μ	σ	$V = \frac{\sigma}{\mu}$	$m_{\epsilon,\%}^{*}$
P(X) 	$\frac{L}{2}$	$\frac{L}{6}$	$\frac{1}{3} = 0.333$	+13.8
P(X) 	$\frac{2}{3}L$	$\frac{2}{3\sqrt{8}}L$	$\frac{1}{\sqrt{8}} = 0.354$	+13.1
P(X) 	$\frac{L}{2}$	$\frac{L}{2\sqrt{6}}$	$\frac{1}{\sqrt{6}} = 0.408$	+11.1
P(X) 	$\frac{L}{2}$	$\frac{L}{2\sqrt{3}}$	$\frac{1}{\sqrt{3}} = 0.577$	+ 3.9
P(X) 	$\frac{L}{2}$	$\frac{\sqrt{11}}{10}L$	$\frac{\sqrt{11}}{5} = 0.663$	0
P(X) 	$\frac{L}{3}$	$\frac{L}{3\sqrt{2}}$	$\frac{1}{\sqrt{2}} = 0.707$	− 2.0
P(X) 	$\frac{L}{2}$	$\frac{L}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}} = 0.707$	− 2.0
P(X) 	$\frac{L}{3}\sqrt{\frac{2}{\pi}}$	$\frac{L}{3}\sqrt{1-\frac{2}{\pi}}$	$\sqrt{\frac{\pi}{2}-1} = 0.756$	− 4.3
P(X) 	$\frac{L}{2}$	$\frac{L}{4}\sqrt{\frac{7}{3}}$	$\frac{1}{2}\sqrt{\frac{7}{3}} = 0.764$	− 4.6

*HERE A POSITIVE ERROR DENOTES A CONSERVATIVE RESULT; A NEGATIVE ERROR, AN OPTIMISTIC RESULT.

small — on the order of 10 percent — and appears to be biased in a conservative direction.

E. EXAMPLE

Given the transfer function

$$G(S) = \frac{a_0 S^2 + a_1 S + a_2}{b_0 S^3 + b_1 S^2 + b_2 S + b_3}$$

where $S \triangleq$ the Laplace operator, find the 3σ variation in its gain characteristic at $\omega = 10$ rad/sec, assuming the 3σ dispersion in each parameter is 10 percent. The 3σ dispersion in gain will be computed in two ways: first by the method of simultaneous tolerancing and then by the RSS method.

Applying Eq. (15) with $k = 3$ and $n = 7$ gives an $m = 1.36$, which corresponds to tolerance deviations of 4.53 percent. Calculation of the nominal and toleranced values of $|G(S)|$ is illustrated in Table 2. (Note that the signs associated with the tolerances were deliberately chosen so that the effects of the tolerances were in the same direction.)

The RSS method is illustrated in Table 3. Here the toleranced values of $|G(S)|$ are obtained by tolerancing the corresponding parameters one at a time, rather than simultaneously as was done previously.

Note that the difference between the two methods is less than 2 percent in this particular example.

Table 2. Computation of a 3σ Dispersion by the Simultaneous Tolerance Technique

Parameter	Nominal Value	Tolerance Applied, %	Toleranced Value
a_0	1	+4.53	1.0453
a_1	20	+4.53	20.906
a_2	75	-4.53	71.6025
b_0	0.1	+4.53	0.1045
b_1	3	-4.53	2.8641
b_2	27.5	-4.53	26.2542
b_3	75	+4.53	78.3975
$\|G(S)\|_{S = j10}$	0.7071		0.8102
$3\sigma_G = G_{tol} - G_{nom} = 0.1031$			

Table 3. Computation of a 3σ Dispersion by the RSS Method

Parameter	Tolerance Applied, %	G_{tol}	ΔG $(G_{tol} - G_{nom})$	$(\Delta G)^2$
a_0	+10	0.7123	0.0052	0.000027
a_1	+10	0.7768	0.0697	0.004858
a_2	-10	0.7108	0.0037	0.000014
b_0	+10	0.7224	0.0153	0.000234
b_1	-10	0.7693	0.0622	0.003869
b_2	-10	0.7492	0.0421	0.001772
b_3	+10	0.7220	0.0149	0.000222
				$\Sigma(\Delta G)^2 = 0.010996$
				$3\sigma_G = \text{RSS} = \sqrt{\Sigma(\Delta G)^2} = 0.1049$

As a matter of interest, the alternate interpretation of the analysis technique is mentioned here. Thus, if 1.36σ parameter deviations were stacked in a worst-on-worst manner it would lead to a 3σ dispersion in $|G(s)|$ equal to 0.1031.

APPENDIX

A DISCUSSION OF THE LOWER AND UPPER BOUNDS OF THE COEFFICIENT OF VARIATION V OF THE TOLERANCE TERMS

If the distribution of the $|K_i \sigma_{X_i}|$ terms were not one-sided, (i.e., if the terms could take on positive and negative values), then V would be completely unbounded. Because the distribution is, in fact, one-sided, this puts a severe constraint on the range of variation of V, as will be noted shortly.

From Eqs. (10) and (11) in the text the square of the coefficient of variation can be expressed as

$$V^2 = \frac{n \sum_{i=1}^{n} x_i^2}{\left(\sum_{i=1}^{n} x_i\right)^2} - 1 \qquad \text{(A-1)*}$$

where

$$x_i > 0$$

The Schwarz inequality for the trivial case of a set of positive numbers states that

$$\left(\sum_{i=1}^{n} x_i\right)^2 \leq n \sum_{i=1}^{n} x_i^2 \qquad \text{(A-2)}$$

* The notation is changed here from that used previously, i.e., $|K_i \sigma_{X_i}|$ to x_i for convenience.

Comparing Eq. (A-1) with Eq. (A-2) it follows that $V \geq 0$ thus establishing the lower bound on the coefficient of variation. (This lower bound can also be arrived at intuitively and corresponds to a set of terms having zero standard deviation and finite mean — which simply means that all the terms are equal.)

An upper bound can be determined by considering the worst case distribution of a set of tolerance terms, i.e., the one that leads to the largest possible value for V. If all of the terms are required to satisfy $x_m \leq x_i \leq x_M$, where x_m is the minimum value of the set and x_M is the maximum value, it can be shown using the Principle of Optimality* that V is maximized when one of the terms takes on the maximum value and all the remaining terms take on the minimum value. Therefore, the configuration

$$x_i = \begin{cases} x_M & i = 1 \\ x_m & i = 2, 3, \ldots, n \end{cases} \tag{A-3}$$

when substituted into Eq. (A-1) leads to the maximum value for the coefficient of variation of

$$V_{max} = \frac{(x_M - x_m)\sqrt{n - 1}}{x_M + (n - 1)\, x_m} \tag{A-4}$$

However, Eq. (A-4) is much too improbable a situation to be encountered in practice since it represents the most severe condition possible based on the

* R. Bellman, Adaptive Control Processes, A Guided Tour, Princeton University Press, Princeton, New Jersey (1961).

the worst possible configuration of tolerance terms. Furthermore, distributions which tend to approach this worst case condition are specifically prohibited on the basis of Assumption 2. In the more general case where n - 2 of the terms do not take on the value x_m but take on other values in the interval between x_m and x_M, much smaller values of V will prevail. After consideration of many random sequences which do not contradict Assumption 2, it appears that a much more realistic upper bound on the coefficient of variation is unity. This corresponds to a distribution where x_m is zero and where half the terms are equal to x_m and the remaining half are equal to x_M.

On the basis of the foregoing heuristic reasoning, the coefficient of variation is taken to be constrained to the region between zero and unity, that is

$$0 \le V \le 1 \qquad \text{(A-5)}$$

That Eq. (A-5) is reasonably valid is borne out in Table 1 in the text where V varied only from 0.333 to 0. [illegible] cases considered. Although this does not constitute proof that this constraint is satisfied in all instances, it does illustrate the compelling tendancy for V to satisfy the constraint of Eq. (A-5) for a wide variety of one-sided distributions. Additionally, fulfillment of Assumption 2 will help assure satisfaction of this constraint in practically all instances.

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13. ABSTRACT

This paper presents a new and simplified method of performing a dispersion analysis. The technique consists of performing a single analysis in which appropriately scaled system tolerances are applied simultaneously in the same direction. This contrasts with the popular and well-known root-sum-square approach, which requires conducting a series of analyses in which the system parameters are varied one at a time and the results are then root-sum-squared. The difference between the proposed approach and the conventional approach is on the order of 10 percent, usually in a conservative direction.

An interesting corollary of the technique is the estimation of the number of standard deviations contained in a dispersion resulting from a number of parameters being displaced from their nominal values in a worst-on-worst fashion.

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14 KEY WORDS
Dispersion Analysis Root-Sum-Square Coefficient of Variation Simultaneous Tolerances Influence Coefficients

Abstract (Continued)